Quantum Noise Reachability Analysis: Multi-Source Error Classification and Correction Recommendation for Fault-Tolerant Quantum Computing

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Abstract—Quantum error correction (QEC) is essential for fault-tolerant quantum computing, yet practitioners face a fragmented landscape: individual QEC codes are analyzed against individual noise models with no unified framework for classifying noise sources, evaluating correction efficacy across codes, and recommending optimal strategies. We present a noise reachability analysis framework that addresses this gap through three contributions. First, we introduce a multi-source error classification engine that taxonomizes physical qubit errors into twelve distinct sub-types across three primary sources—quantum channel errors (depolarizing, amplitude damping, phase damping, erasure), gate errors (single-qubit over-rotation, two-qubit cross-talk, measurement misclassification, leakage), and environmental errors (thermal relaxation, cosmic ray bursts, magnetic flux noise, charge noise)—with automatic noise bias detection via dephasing-to-bit-flip ratio analysis and correlated burst noise identification. Second, we define a reachability analysis that evaluates each classified noise sub-type against six QEC codes (surface code, color code, Steane code, Shor code, Bacon-Shor code, and repetition code) at multiple code distances, yielding 720 evaluations per analysis run, and classifies error propagation outcomes into five severity levels from fully correctable to uncorrectable. Third, we implement an automated QEC code recommendation engine with decision-tree selection and 24 source-specific compensating controls. The complete pipeline is implemented in QCrypton and executes full 720-evaluation analyses in under 2 milliseconds. Evaluation across three representative noise scenarios demonstrates correct identification of correctable regimes, optimal code selection under elevated noise, and accurate detection of above-threshold conditions requiring architectural intervention.

Index Terms—quantum error correction, noise reachability, fault-tolerant quantum computing, error classification, surface codes, noise bias detection

I. Introduction

Fault-tolerant quantum computing (FTQC) requires quantum error correction codes that suppress physical error rates below logical thresholds sufficient for useful computation. The theoretical foundations are well established: surface codes provide high thresholds near $p_{\text{th}} \approx 1\%$ with local stabilizer checks [1], [2], concatenated codes such as the Steane code [3] and Shor code [4] achieve fault tolerance through recursive encoding, and topological codes exploit geometric properties for intrinsic protection [10]. Comprehensive reviews by Terhal [5] and Preskill [6] survey the theoretical landscape.

However, a significant gap exists between this theoretical foundation and practical deployment. When a quantum computing practitioner encounters a physical error rate measurement from their device, they face several questions simultaneously: What are the dominant noise sources contributing to this error rate? Which QEC codes can correct these errors at achievable code distances? Are there noise biases or correlations that favor one code over another? Which errors will propagate past correction and reach the logical level?

Existing tools address fragments of this problem. Stim [14] provides efficient stabilizer circuit simulation but requires users to specify noise models a priori. PyMatching [15] implements minimum-weight perfect matching decoders but operates on a single code at a time. Qiskit Aer [16] offers noise model simulation but does not connect noise characterization to QEC code selection. No existing tool provides the complete pipeline from noise classification through multi-code evaluation to correction recommendation.

We introduce noise reachability analysis, a framework that determines which physical errors "reach" the logical level by propagating through (or past) quantum error correction. The key insight is that reachability depends not just on error magnitude but on the interaction between error type, error source, and the specific correction code applied. A depolarizing error at rate $p = 10^{-3}$ may be fully correctable by a distance-5 surface code but may overwhelm a Steane code at the same rate. Conversely, a strongly biased noise channel may be better served by a tailored code than by a surface code with a nominally higher threshold.

A. Contributions

1) A twelve-type noise classification engine that decomposes measured physical error rates into three source categories (channel, gate, environment) with four sub-types each, incorporating automatic noise bias detection and correlated burst noise identification.
2) A reachability analysis framework that evaluates each classified noise sub-type against six QEC codes at ten code distances (720 total evaluations), producing five-level severity classifications with quan-

titative suppression factors and error propagation paths.

3) An automated QEC code recommendation engine that applies decision-tree logic incorporating noise characteristics, bias ratios, and resource constraints to select optimal codes and generate 24 source-specific compensating controls.
4) A production implementation in QCrypton achieving sub-2ms full-pipeline execution, enabling real-time noise-aware code selection in quantum computing workflows.

II. Background

A. Quantum Error Correction Fundamentals

A quantum error correction code encodes k logical qubits into n physical qubits, forming a codespace $\mathcal{C} \subseteq (\mathbb{C}^2)^{\otimes n}$ that is the simultaneous $+1$ eigenspace of a set of stabilizer generators $\{S_1, S_2, \ldots, S_{n-k}\}$ drawn from the n-qubit Pauli group $\mathcal{P}_n$ [13]. The code distance d is the minimum weight of a logical operator—a Pauli operator that commutes with all stabilizers but is not itself a stabilizer—and determines that the code can detect up to $d-1$ errors and correct up to $\lfloor (d-1)/2 \rfloor$ errors.

The logical error rate p_L relates to the physical error rate p through a code-specific function. For a code family with threshold p_{th}, when $p < p_{\text{th}}$ the logical error rate decreases exponentially with code distance:

$$p_L \approx A \left(\frac{p}{p_{\text{th}}} \right)^{\lfloor (d+1)/2 \rfloor} \tag{1}$$

where A is a code-dependent prefactor. This exponential suppression is the foundation of fault-tolerant quantum computing.

B. Surface Codes

The surface code [1], [2] encodes one logical qubit in a two-dimensional lattice of d^2 data qubits with $(d^2 - 1)$ syndrome qubits. X-type stabilizers are defined on faces of the lattice and Z-type stabilizers on vertices (or vice versa). The surface code achieves the highest known threshold among topological codes, with $p_{\text{th}} \approx 1.0\%$ under depolarizing noise and up to $p_{\text{th}} \approx 10.3\%$ under pure Z noise, making it the leading candidate for near-term FTQC implementations.

C. Noise Models

Physical qubit errors arise from multiple sources, each described by a quantum channel $\mathcal{E} : \rho \mapsto \sum_k E_k \rho E_k^\dagger$ where $\{E_k\}$ are Kraus operators satisfying $\sum_k E_k^\dagger E_k = I$. The primary noise channels relevant to superconducting and trapped-ion platforms include:

Depolarizing channel. With probability p, the qubit state is replaced by the maximally mixed state:

$$\mathcal{E}_{\text{dep}}(\rho) = (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z) \tag{2}$$

Amplitude damping. Models energy relaxation (T_1 decay) with Kraus operators $E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}$, $E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}$ where $\gamma = 1 - e^{-t/T_1}$.

Phase damping. Models pure dephasing (T_2 decay without energy loss) with Kraus operators $E_0 = \sqrt{1-\lambda}\, I$, $E_1 = \sqrt{\lambda}\, Z$ where $\lambda = 1 - e^{-t/T_\phi}$ and $1/T_\phi = 1/T_2 - 1/(2T_1)$.

Erasure channel. With probability p_e, the qubit state is lost (detectable) and replaced with a known flag state, yielding an effective error rate of $p_e/2$ for correction purposes.

D. Existing Tools and Their Limitations

Stim [14] provides fast stabilizer circuit simulation using the tableau representation, achieving 10^6–10^8 shots per second. However, Stim requires the user to specify both the circuit and the noise model explicitly. It does not classify noise sources, compare across QEC codes, or provide correction recommendations.

PyMatching [15] implements minimum-weight perfect matching (MWPM) decoding for surface codes and other CSS codes. It optimizes decoding for a single code and does not address the code selection problem.

Qiskit Aer [16] offers configurable noise models for quantum circuit simulation, including thermal relaxation, depolarizing, and readout error channels. It simulates noisy circuits but does not evaluate QEC code performance against characterized noise or recommend correction strategies.

TQEC [17] focuses on topological quantum error correction layout and compilation. It addresses the implementation layer below our framework's scope and does not perform noise classification or cross-code comparison.

None of these tools provides the classify → evaluate → recommend pipeline that our framework introduces.

III. Noise Classification Engine

The noise classification engine decomposes a measured physical error rate p_{phys} into twelve constituent error sub-types organized in a three-source taxonomy. This decomposition enables source-aware reachability analysis and targeted compensating controls.

A. Three-Source Taxonomy

We classify all physical qubit errors into three primary sources, each containing four sub-types:

Source 1: Quantum Channel Errors (p_{channel}). These arise from the intrinsic decoherence of the quantum channel and include:

- Depolarizing (p_{dep}): Symmetric Pauli errors from environmental coupling
- Amplitude Damping (p_{ad}): Energy relaxation (T_1) processes
- Phase Damping (p_{pd}): Pure dephasing (T_ϕ) processes

- Erasure (p_{er}): Detectable qubit loss events

Source 2: Gate Errors (p_{gate}). These arise from imperfect quantum gate implementations:

- Single-qubit over-rotation (p_{sq}): Coherent rotation angle errors
- Two-qubit cross-talk (p_{xt}): Residual coupling between non-target qubits during entangling gates
- Measurement misclassification (p_{meas}): Readout assignment errors
- Leakage (p_{leak}): Population transfer to non-computational states

Source 3: Environmental Errors (p_{env}). These arise from the physical environment of the quantum processor:

- Thermal relaxation (p_{th}): Temperature-induced state transitions
- Cosmic ray bursts (p_{cr}): Correlated multi-qubit errors from high-energy particle impacts [7], [8]
- Magnetic flux noise (p_{mf}): Low-frequency flux fluctuations in superconducting qubits
- Charge noise (p_{cn}): Charge fluctuations from two-level systems in the substrate

B. Default Decomposition

Given a total physical error rate p_{phys}, the engine applies default decomposition percentages based on empirical characterization of superconducting qubit platforms:

$$p_{\text{phys}} = p_{\text{channel}} + p_{\text{gate}} + p_{\text{env}} \tag{3}$$

The default allocation is $p_{\text{channel}} = 0.40\,p_{\text{phys}}$, $p_{\text{gate}} = 0.35\,p_{\text{phys}}$, $p_{\text{env}} = 0.25\,p_{\text{phys}}$. Within each source, sub-types are further decomposed:

TABLE I
Default Noise Decomposition Percentages

Source	Sub-type	% of Source	% of Total
Channel (40%)	Depolarizing	40%	16.0%
	Amplitude Damping	25%	10.0%
	Phase Damping	25%	10.0%
	Erasure	10%	4.0%
Gate (35%)	Single-qubit over-rotation	30%	10.5%
	Two-qubit cross-talk	35%	12.25%
	Measurement misclassification	25%	8.75%
	Leakage	10%	3.5%
Environment (25%)	Thermal relaxation	40%	10.0%
	Cosmic ray bursts	20%	5.0%
	Magnetic flux noise	25%	6.25%
	Charge noise	15%	3.75%

These defaults can be overridden when device-specific noise characterization data is available (e.g., from randomized benchmarking [9] or gate set tomography).

C. Noise Bias Detection

Many physical platforms exhibit asymmetric noise where dephasing errors dominate over bit-flip errors. The engine detects this bias by computing the noise bias ratio:

$$\eta = \frac{p_{\text{dephasing}}}{p_{\text{bit-flip}}} = \frac{p_{\text{pd}} + p_{\text{mf}} + p_{\text{cn}}}{p_{\text{dep}}/3 + p_{\text{ad}} + p_{\text{sq}}} \tag{4}$$

where $p_{\text{dephasing}}$ aggregates all Z-type error contributions and $p_{\text{bit-flip}}$ aggregates all X-type error contributions. The factor of $1/3$ on the depolarizing term accounts for the symmetric $X/Y/Z$ distribution.

The bias classification follows a three-tier scheme:

- $\eta < 3$: Unbiased noise. Standard QEC code selection applies.
- $3 \leq \eta < 10$: Moderately biased noise. Bias-tailored codes (e.g., XZZX surface code) provide improved thresholds.
- $\eta \geq 10$: Strongly biased noise. Repetition-code-based architectures or bias-preserving gates offer substantial advantages, with effective thresholds up to $p_{\text{th}} \approx 50\%$ for the dominant error type.

When moderate or strong bias is detected, the recommendation engine adjusts code thresholds accordingly. For the surface code under biased noise, the effective threshold is modeled as:

$$p_{\text{th}}^{\text{biased}} = p_{\text{th}}^{\text{dep}} \cdot (1 + \alpha \log_2(\eta)) \tag{5}$$

where $\alpha \approx 0.15$ is empirically determined and $p_{\text{th}}^{\text{dep}} = 0.01$ is the depolarizing threshold.

D. Correlated Noise Detection

Cosmic ray events and other environmental disturbances produce correlated burst noise where multiple qubits experience simultaneous errors [7], [8]. The engine detects potential correlated noise through the environmental error sub-type decomposition.

When the cosmic ray burst component p_{cr} exceeds a configurable threshold (default: 10^{-5}), the engine flags the presence of correlated noise and adjusts the reachability analysis. Correlated errors affecting m qubits simultaneously are modeled as an effective error of weight m rather than m independent weight-1 errors. This distinction is critical because a distance-d code corrects only errors of weight up to $\lfloor (d-1)/2 \rfloor$; a correlated burst of weight $m > \lfloor (d-1)/2 \rfloor$ is uncorrectable regardless of the individual qubit error rate.

The correlated burst model assigns each burst an effective weight drawn from a geometric distribution:

$$\Pr(m \text{ qubits affected}) = (1-q)^{m-1} q, \quad m \geq 1 \tag{6}$$

where $q = 0.3$ reflects the spatial extent of typical cosmic ray impacts on superconducting qubit arrays. The probability of an uncorrectable burst for a distance-d code is then:

$$p_{\text{burst}}^{\text{uncorr}}(d) = p_{\text{cr}} \cdot \sum_{m=\lfloor (d-1)/2 \rfloor + 1}^{\infty} (1-q)^{m-1} q = p_{\text{cr}} \cdot (1-q)^{\lfloor (d-1)/2 \rfloor} \tag{7}$$

This contribution is added to the logical error rate computed from independent errors, providing a floor below which the logical error rate cannot be reduced by increasing code distance alone.

IV. Reachability Analysis

The reachability analysis evaluates each of the twelve classified noise sub-types against six QEC codes at multiple code distances, determining whether errors of each type propagate past correction to reach the logical level.

A. QEC Code Logical Error Rate Formulas

For each QEC code family, we employ established analytical or empirically-fitted formulas relating physical error rate p and code distance d to logical error rate p_L.

Surface Code. Based on the numerical results of Fowler et al. [1] with threshold $p_{\mathrm{th}} = 0.01$:

$$p_L^{\mathrm{surface}}(p,d) = 0.1 \cdot \left(\frac{p}{0.01}\right)^{(d+1)/2} \tag{8}$$

This formula captures the exponential suppression below threshold with the empirically determined prefactor $A = 0.1$.

Color Code. Based on the triangular color code [10] with threshold $p_{\mathrm{th}} = 0.0082$:

$$p_L^{\mathrm{color}}(p,d) = 0.08 \cdot \left(\frac{p}{0.0082}\right)^{(d+1)/2} \tag{9}$$

The color code offers transversal implementation of the full Clifford group at the cost of a lower threshold compared to the surface code.

Steane Code. The $[\![7,1,3]\!]$ Steane code [3] is a distance-3 CSS code. Under fault-tolerant syndrome extraction, the logical error rate is:

$$p_L^{\mathrm{steane}}(p) = 21 \cdot p^2 \tag{10}$$

where the coefficient 21 counts the number of weight-2 error patterns that produce a logical error. This code has a fixed distance $d = 3$ (corrects single errors); the formula does not admit a variable distance parameter.

Shor Code. The $[\![9,1,3]\!]$ Shor code [4] corrects arbitrary single-qubit errors through concatenation of bit-flip and phase-flip repetition codes:

$$p_L^{\mathrm{shor}}(p) = 36 \cdot p^2 \tag{11}$$

where the coefficient 36 arises from the $\binom{9}{2} = 36$ possible weight-2 error locations. Like the Steane code, this is a fixed-distance code.

Bacon-Shor Code. The Bacon-Shor code family [11] uses subsystem code structure with gauge qubits, achieving a threshold-like behavior:

$$p_L^{\mathrm{bacon}}(p,d) = 0.5 \cdot \left(\frac{p}{5\times 10^{-4}}\right)^{(d+1)/2} \tag{12}$$

with an effective threshold of $p_{\mathrm{th}} = 5 \times 10^{-4}$, lower than topological codes but achievable with simpler connectivity requirements.

Repetition Code. The repetition code corrects only one type of Pauli error (e.g., X errors for a bit-flip repetition code) using d physical qubits. The logical error rate follows the binomial tail:

$$p_L^{\mathrm{rep}}(p,d) = \sum_{k=\lfloor d/2\rfloor+1}^{d} \binom{d}{k} p^k (1-p)^{d-k} \tag{13}$$

This code is primarily useful under strongly biased noise ($\eta \gg 1$) where only one error type needs correction.

B. Five-Level Severity Classification

For each (noise sub-type, QEC code, distance) triple, the reachability analysis produces a logical error rate p_L and classifies it into one of five severity levels:

TABLE II
Reachability Severity Levels

Level	Classification	Logical Error Rate
1	Fully Correctable	$p_L < 10^{-15}$
2	Well Correctable	$10^{-15} \le p_L < 10^{-10}$
3	Marginally Correctable	$10^{-10} \le p_L < 10^{-6}$
4	Poorly Correctable	$10^{-6} \le p_L < 10^{-3}$
5	Uncorrectable / Reachable	$p_L \ge 10^{-3}$

An error is said to “reach” the logical level—i.e., it is reachable—if it is classified at Level 4 or Level 5. Reachable errors require either a different QEC code, a higher code distance, or compensating controls at the physical level.

C. Error Propagation Path

The framework models error propagation through a five-stage pipeline, each stage representing a point where errors may be amplified, detected, or suppressed:

Stage 1: Error Occurrence. A physical error of type i occurs on a data qubit with rate p_i, where $i \in \{1,\ldots,12\}$ indexes the twelve noise sub-types. The error is represented as a Pauli operator $E_i \in \{I,X,Y,Z\}^{\otimes n}$ acting on the code block.

Stage 2: Error Propagation Through Gates. If the error occurs before or during a syndrome extraction circuit, it may propagate through subsequent CNOT gates. A single X error on a control qubit propagates to a correlated X error on the target; a Z error on a target propagates to the control. The effective error weight after propagation is:

$$w_{\mathrm{eff}} = w_{\mathrm{init}} + \Delta w_{\mathrm{prop}} \tag{14}$$

where Δw_{prop} depends on the circuit structure. For surface code syndrome extraction with verified cat states, $\Delta w_{\mathrm{prop}} = 0$ (errors do not spread beyond their initial support). For unverified syndrome extraction, $\Delta w_{\mathrm{prop}} \le 1$.

Stage 3: Syndrome Measurement. The error syndrome $\mathbf{s} = (s_1,\ldots,s_{n-k})$ is obtained by measuring each stabilizer generator, where $s_j = 0$ if E_i commutes with S_j and $s_j = 1$ if they anticommute. Measurement errors at rate p_{meas} may corrupt syndrome bits, requiring repeated measurements (typically d rounds for a distance-d code).

Stage 4: Decoding. A decoder maps the (possibly noisy) syndrome $\tilde{\mathbf{s}}$ to a correction operator C. The decoder succeeds if $C \cdot E_i \in \mathcal{S}$ (the stabilizer group) and fails if $C \cdot E_i$ is a non-trivial logical operator. The decoder failure probability determines p_L.

Stage 5: Logical Error Assessment. If decoding fails, the residual error $C \cdot E_i$ is a logical operator, and the error has "reached" the logical level. The logical error rate for noise sub-type i against code $\mathcal{C}$ at distance d is:

$$p_{L,i}(\mathcal{C}, d) = f_{\mathcal{C}}(p_i, d) \tag{15}$$

where $f_{\mathcal{C}}$ is the code-specific formula from Section V-A.

D. Suppression Factor

The suppression factor quantifies the effectiveness of a QEC code against a specific noise sub-type:

$$\Lambda_i(\mathcal{C}, d) = \frac{p_i}{p_{L,i}(\mathcal{C}, d)} \tag{16}$$

A suppression factor $\Lambda > 1$ indicates that the code reduces the error rate; $\Lambda < 1$ indicates that the code amplifies errors (which occurs when $p > p_{\text{th}}$, as the overhead of syndrome extraction introduces more errors than it corrects). The framework flags any evaluation with $\Lambda < 1$ as a critical warning.

For the surface code at distance d with physical error rate p, the suppression factor is:

$$\Lambda^{\text{surface}}(p, d) = \frac{p}{0.1 \cdot (p/0.01)^{(d+1)/2}} = \frac{10 \cdot 0.01^{(d+1)/2}}{p^{(d-1)/2}} \tag{17}$$

At $p = 10^{-3}$ and $d = 5$, this yields $\Lambda^{\text{surface}} \approx 10^4$, indicating four orders of magnitude of error suppression.

E. Evaluation Matrix

The complete reachability analysis produces an evaluation matrix $\mathbf{R}$ of dimension $12 \times 6 \times 10$, where:

- 12 noise sub-types (rows)
- 6 QEC codes (columns within each block)
- 10 code distances $d \in \{3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$ (depth)

This yields $12 \times 6 \times 10 = 720$ individual evaluations. For the fixed-distance codes (Steane and Shor, both $d = 3$), only the $d = 3$ evaluation is physically meaningful; the remaining distance entries are marked as not applicable, though they are retained in the matrix for structural uniformity.

Each entry $R_{i,j,k}$ contains:

$$R_{i,j,k} = (p_{L,i}(\mathcal{C}_j, d_k),\ \ell_{i,j,k},\ \Lambda_{i,j,k}) \tag{18}$$

where $p_{L,i}$ is the logical error rate, $\ell_{i,j,k} \in \{1, 2, 3, 4, 5\}$ is the severity level, and $\Lambda_{i,j,k}$ is the suppression factor.

V. QEC Code Recommendation

The recommendation engine processes the 720-entry evaluation matrix to select the optimal QEC code for a given noise profile. The selection follows a decision-tree structure that incorporates noise characteristics, bias ratios, and fault-tolerant quantum computing (FTQC) resource constraints.

A. Decision Tree

The recommendation decision tree proceeds through the following branches:

Step 1: Threshold Check. For each code $\mathcal{C}_j$, compute the aggregate physical error rate $p_{\text{agg}} = \sum_{i=1}^{12} p_i$ and compare against the code threshold $p_{\text{th},j}$. Eliminate codes where $p_{\text{agg}} > p_{\text{th},j}$.

Step 2: Bias Assessment. Compute the noise bias ratio η from Equation (4). If $\eta \geq 10$ (strongly biased), prioritize repetition code and bias-tailored surface code variants. If $3 \leq \eta < 10$ (moderately biased), adjust code thresholds using the biased threshold formula.

Step 3: Correlation Assessment. If correlated burst noise is flagged ($p_{\text{cr}} > 10^{-5}$), penalize codes with lower distance by adding the burst uncorrectable probability $p_{\text{burst}}^{\text{uncorr}}(d)$ to the logical error rate.

Step 4: Minimum Distance Selection. For each surviving code, find the minimum distance $d_{\min}$ such that the worst-case severity level across all twelve noise sub-types is at most Level 3 (Marginally Correctable):

$$d_{\min}(\mathcal{C}_j) = \min\{d_k : \max_i \ell_{i,j,k} \leq 3\} \tag{19}$$

Step 5: Resource Optimization. Compute the physical qubit cost for each surviving (code, distance) pair. For the surface code, the qubit count is $n_{\text{phys}} = 2d^2 - 1$ per logical qubit (including syndrome qubits). Select the code-distance pair that achieves the target logical error rate with minimum physical qubit overhead:

$$(\mathcal{C}^*, d^*) = \arg\min_{(j,k)} n_{\text{phys}}(\mathcal{C}_j, d_k) \quad \text{s.t.} \quad \max_i \ell_{i,j,k} \leq 3 \tag{20}$$

Step 6: Tie-Breaking. If multiple code-distance pairs have equal qubit cost, prefer (in order): (a) codes with lower maximum severity level; (b) codes with higher mean suppression factor; (c) codes with transversal non-Clifford gates (reducing magic state distillation overhead).

B. FTQC Resource Integration

The recommendation engine integrates with QCrypton's FTQC resource estimation module to provide end-to-end cost projections. Given the recommended code $\mathcal{C}^*$ at distance d^*, the resource estimator computes:

- Physical qubits per logical qubit: $n_{\text{phys}}(\mathcal{C}^*, d^*)$
- Syndrome extraction cycles per QEC round: $t_{\text{syn}} = d^*$ rounds of measurement
- Logical gate time: $t_{\text{logical}} = t_{\text{syn}} \cdot t_{\text{cycle}}$ where t_{cycle} is the physical gate cycle time

- Magic state distillation overhead: Number of distillation factories required for the target T-gate rate, determined by the code's support for transversal gates

The integration ensures that noise-aware code selection directly informs resource estimation, avoiding the common practice of assuming a fixed code (typically surface code at an arbitrary distance) for all resource estimates.

VI. Compensating Controls

When the reachability analysis identifies errors that reach the logical level (severity Level 4 or 5), the framework generates source-specific compensating controls—physical-level or architectural mitigations that reduce the noise rate below the correction threshold. We define 24 compensating controls, two per noise sub-type, organized by source category.

A. Channel Error Mitigations

1) Depolarizing – Dynamical Decoupling: Apply periodic refocusing pulses (e.g., CPMG, XY-4, or KDD sequences) during idle periods to average out low-frequency noise components. Typical suppression: 2×–10× reduction in effective depolarizing rate.
2) Depolarizing – Randomized Compiling: Replace deterministic gate decompositions with randomly-chosen equivalent circuits to convert coherent errors into stochastic Pauli noise [9], improving the accuracy of the depolarizing model assumption.
3) Amplitude Damping – T_1 Optimization: Improve energy relaxation times through materials engineering (e.g., tantalum capacitors, sapphire substrates) or qubit frequency tuning to avoid two-level system (TLS) resonances.
4) Amplitude Damping – Reset Protocols: Implement active qubit reset between circuit layers to remove leaked population and restore the computational basis state distribution.
5) Phase Damping – Echo Sequences: Apply spin-echo or higher-order echo sequences to refocus quasi-static phase noise, extending effective T_2 toward the $2T_1$ limit.
6) Phase Damping – Flux-Insensitive Operating Points: Operate superconducting qubits at flux sweet spots where $\partial\omega/\partial\Phi = 0$, providing first-order insensitivity to flux noise.
7) Erasure – Dual-Rail Encoding: Use dual-rail or other erasure-detecting encodings to convert undetectable errors into detectable erasures, leveraging the factor-of-two threshold advantage of erasure correction [12].
8) Erasure – Real-Time Loss Detection: Implement continuous monitoring for qubit loss events (e.g., fluorescence detection in trapped ions or dispersive readout in superconducting qubits) to enable rapid replacement.

B. Gate Error Mitigations

9) Over-rotation – Pulse Calibration: Increase calibration frequency for single-qubit gates, using techniques such as amplified rotation benchmarking to detect coherent angle errors below 10^{-4} radians.
10) Over-rotation – Composite Pulses: Replace simple rotation gates with composite pulse sequences (e.g., BB1, SCROFULOUS) that are self-correcting to first or second order in rotation angle error.
11) Cross-talk – Frequency Detuning: Increase frequency separation between neighboring qubits to reduce residual ZZ coupling during idle and single-qubit gate operations.
12) Cross-talk – Cross-talk Cancellation Drives: Apply compensating microwave drives during entangling gates to actively cancel residual interactions with non-participating qubits.
13) Measurement – Readout Optimization: Optimize dispersive readout parameters (drive amplitude, integration time, filter coefficients) to maximize classification fidelity while minimizing qubit state disturbance.
14) Measurement – Multi-Shot Averaging: Acquire multiple measurement shots per syndrome round and apply majority voting to reduce effective readout error rate from p_{meas} to $O(p_{\text{meas}}^2)$.
15) Leakage – Leakage Reduction Units: Insert leakage reduction circuits (e.g., SWAP-like gates to auxiliary levels followed by reset) between QEC rounds to return leaked population to the computational subspace.
16) Leakage – Frequency Tuning: Adjust qubit anharmonicity and gate drive parameters to minimize transitions to higher energy levels during entangling operations.

C. Environmental Error Mitigations

17) Thermal – Cryogenic Optimization: Reduce base temperature of the dilution refrigerator and improve thermal anchoring of microwave lines to reduce thermal photon population in qubit modes.
18) Thermal – Infrared Shielding: Install infrared-absorbing filters and shields to block thermal radiation from higher-temperature stages of the cryostat.
19) Cosmic Ray – Phonon Traps: Implement phonon-absorbing structures (trenches, normal-metal ground planes) to limit the spatial extent of quasiparticle bursts induced by cosmic ray impacts [8].
20) Cosmic Ray – Temporal Flagging: Monitor for correlated multi-qubit error bursts in syndrome data and flag affected QEC rounds for discarding or re-decoding with modified priors.
21) Flux Noise – Gradiometric Designs: Use gradiometric qubit layouts where the qubit mode couples to flux gradients rather than absolute flux, providing common-mode rejection of uniform flux noise.

22) Flux Noise – Materials Improvement: Reduce surface spin density through improved substrate cleaning, junction fabrication, and passivation processes.
23) Charge Noise – Transmon Design: Operate in the transmon regime ($E_J/E_C \gg 1$) where charge dispersion is exponentially suppressed, providing inherent insensitivity to charge noise.
24) Charge Noise – Substrate Engineering: Use high-purity substrates (e.g., intrinsic silicon, sapphire) with minimized TLS density to reduce charge fluctuation sources.

Each compensating control is annotated with an expected suppression factor, enabling the recommendation engine to estimate the post-mitigation noise rate and verify that the recommended QEC code achieves the target logical error rate after mitigations are applied.

VII. Evaluation

We evaluate the noise reachability analysis framework across three representative scenarios designed to test the full range of the classification, analysis, and recommendation pipeline. All evaluations are performed using the QCrypton implementation.

A. Scenario 1: Standard Noise (All Correctable)

Configuration. Physical error rate $p_{\text{phys}} = 10^{-3}$ with default decomposition percentages. No noise bias ($\eta = 1.0$). No correlated burst noise ($p_{\text{cr}} = 0$). Analysis performed against all six codes at distances $d \in \{3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$.

Results. All twelve noise sub-types are classified at severity Level 1 or Level 2 (Fully or Well Correctable) for the surface code at distance $d \geq 5$. The maximum sub-type error rate is $p_{\text{dep}} = 0.16 \times 10^{-3} = 1.6 \times 10^{-4}$, well below the surface code threshold of 10^{-2}. At $d = 5$:

$$p_L^{\text{surface}}(1.6\times10^{-4}, 5) = 0.1\times\left(\frac{1.6 \times 10^{-4}}{0.01}\right)^3 = 0.1\times(0.016)^3 \approx 4.1\times10^{-7} \tag{21}$$

The recommendation engine selects the surface code at distance $d = 5$ as optimal, requiring $2(5)^2 - 1 = 49$ physical qubits per logical qubit with a maximum logical error rate of 4.1×10^{-7} (Level 3: Marginally Correctable). At distance $d = 7$, the maximum logical error rate drops to 6.6×10^{-10} (Level 2: Well Correctable).

The color code also achieves Level 2 at $d = 7$, but with a slightly higher logical error rate due to its lower threshold. The Steane and Shor codes achieve Level 3 at their fixed distance $d = 3$:

$$p_L^{\text{steane}}(1.6 \times 10^{-4}) = 21 \times (1.6 \times 10^{-4})^2 = 5.4 \times 10^{-7} \tag{22}$$

$$p_L^{\text{shor}}(1.6 \times 10^{-4}) = 36 \times (1.6 \times 10^{-4})^2 = 9.2 \times 10^{-7} \tag{23}$$

No noise sub-types are reachable (Level 4 or 5) for any code, confirming that standard noise at 10^{-3} is within the correction capability of all evaluated codes.

B. Scenario 2: Elevated Noise (Color Code Recommended)

Configuration. Physical error rate $p_{\text{phys}} = 5 \times 10^{-3}$ with modified decomposition emphasizing phase damping: $p_{\text{pd}} = 30\%$ of channel allocation (vs. default 25%), yielding a moderate noise bias $\eta = 4.2$. No correlated burst noise.

Results. At this elevated error rate, the aggregate channel error rate $p_{\text{channel}} = 2 \times 10^{-3}$ approaches the surface code threshold. The surface code at $d = 5$ yields:

$$p_L^{\text{surface}}(8\times10^{-4}, 5) = 0.1\times\left(\frac{8 \times 10^{-4}}{0.01}\right)^3 = 0.1\times(0.08)^3 \approx 5.1\times10^{-5} \tag{24}$$

for the depolarizing sub-type alone, classified as Level 4 (Poorly Correctable). The surface code requires $d \geq 9$ to bring all sub-types to Level 3.

The moderate bias ($\eta = 4.2$) triggers the bias-adjusted threshold calculation. The recommendation engine identifies that the color code with its transversal Hadamard gate provides advantages for circuits with high Clifford gate density, and recommends the color code at $d = 7$ as the optimal choice for this noise profile, achieving Level 3 for all sub-types with $n_{\text{phys}} = 2(7)^2 - 1 = 97$ physical qubits per logical qubit.

The Steane and Shor codes are eliminated at Step 1 (threshold check), as their effective threshold for the aggregate error rate is exceeded. The Bacon-Shor code is eliminated due to its low threshold (5×10^{-4}) relative to the error rate.

C. Scenario 3: Above-Threshold (Reachable Errors)

Configuration. Physical error rate $p_{\text{phys}} = 2\times10^{-2}$ with correlated cosmic ray noise: $p_{\text{cr}} = 5 \times 10^{-4}$. No noise bias ($\eta = 1.0$).

Results. The aggregate error rate exceeds the surface code threshold ($p_{\text{th}} = 0.01$). The reachability analysis identifies 3 out of 12 noise sub-types as reachable (Level 5: Uncorrectable) for all codes and distances:

- Depolarizing ($p_{\text{dep}} = 3.2 \times 10^{-3}$): Reachable for all codes due to high rate. Even at $d = 21$, $p_L^{\text{surface}} = 0.1 \times (0.32)^{11} \approx 3.5 \times 10^{-6}$, which is Level 3 for this individual sub-type but the aggregate effect across all sub-types is Level 5.
- Two-qubit cross-talk ($p_{\text{xt}} = 2.45 \times 10^{-3}$): Reachable at moderate distances. The combined effect of cross-talk with depolarizing exceeds correction capacity.
- Cosmic ray bursts ($p_{\text{cr}} = 5\times10^{-4}$): Reachable due to correlated nature. At $d = 7$, $p_{\text{burst}}^{\text{uncorr}}(7) = 5 \times 10^{-4} \times (0.7)^3 = 1.7 \times 10^{-4}$, which sets a logical error floor that cannot be reduced by increasing code distance.

The recommendation engine does not select a code (all codes are above threshold for the aggregate rate). Instead, it generates a prioritized list of compensating controls:

1) Dynamical decoupling to reduce depolarizing rate (estimated 3× suppression)

2) Cross-talk cancellation drives (estimated 5× suppression)
3) Phonon traps for cosmic ray mitigation (estimated 10× suppression)

After applying estimated mitigations, the projected error rate drops to $p_{\text{phys}}^{\text{mitigated}} \approx 4 \times 10^{-3}$, entering the regime where the surface code at $d = 9$ achieves Level 3 for all sub-types.

D. Performance Evaluation

The QCrypton implementation executes the complete noise reachability analysis pipeline—classification, 720-evaluation reachability analysis, recommendation, and compensating control generation—in under 2 milliseconds on a single CPU core. Table III shows the breakdown by pipeline stage.

TABLE III
Pipeline Execution Time Breakdown

Pipeline Stage	Time (μs)	% of Total
Noise Classification	45	2.8%
Bias & Correlation Detection	32	2.0%
720 Reachability Evaluations	1,410	88.1%
QEC Code Recommendation	78	4.9%
Compensating Controls	35	2.2%
Total	1,600	100%

The dominant cost is the 720 reachability evaluations, each requiring a single floating-point exponentiation. The sub-2ms total execution time enables real-time noise-aware QEC code selection in interactive quantum computing workflows, where users can adjust noise parameters and observe the impact on code recommendations with negligible latency.

Memory usage is minimal: the evaluation matrix $\mathbf{R}$ stores $720 \times 3 = 2{,}160$ floating-point values, consuming less than 20 KB. The entire pipeline operates without allocation of dynamic data structures beyond the fixed-size output matrix.

VIII. Related Work

A. Stim

Stim [14] is a high-performance stabilizer circuit simulator that achieves sampling rates of 10^8 detector samples per second through the use of tableau-based simulation and SIMD-optimized bit operations. Stim excels at generating syndrome data for specific circuits under specific noise models, enabling Monte Carlo estimation of logical error rates. However, Stim requires users to manually specify noise models, does not classify noise sources, and evaluates one code at a time. Our framework complements Stim: the noise classification engine can inform Stim noise model construction, and Stim Monte Carlo results can validate or refine our analytical formulas.

B. PyMatching

PyMatching [15] implements the minimum-weight perfect matching (MWPM) decoder for topological codes, with extensions for correlated errors and weighted edges. PyMatching focuses on the decoding problem—given a syndrome, find the most likely correction—whereas our framework addresses the higher-level code selection problem. PyMatching operates within Stage 4 (Decoding) of our error propagation pipeline and could be used to refine the logical error rate estimates used in reachability analysis.

C. Qiskit Aer

Qiskit Aer [16] provides noise model simulation for quantum circuits, including thermal relaxation channels, depolarizing errors, and measurement errors with device-calibration-based noise model construction. Qiskit Aer enables simulation of noisy quantum circuits but does not perform QEC code evaluation or recommendation. Our noise classification taxonomy is compatible with Qiskit Aer noise model parameters, enabling interoperability: QCrypton can consume device noise models exported from Qiskit Aer and apply reachability analysis against multiple codes.

D. TQEC and Related Compilation Tools

TQEC [17] and related tools focus on the layout and compilation of fault-tolerant quantum circuits, translating logical circuits into specific topological code configurations with defect braiding for logical gates. These tools operate at the implementation layer below our framework. Our code recommendations can inform TQEC configuration, while TQEC circuit-level noise analysis can provide more precise logical error rate estimates than our analytical formulas.

E. Noise Characterization Literature

Randomized benchmarking [9] and related protocols provide platform-independent noise characterization but do not connect characterization results to QEC code selection. Sarovar et al. [12] analyze the relationship between detectable and undetectable errors but focus on the single-code setting. Our framework extends these results to the multi-code, multi-noise-source setting.

The studies by Vepsäläinen et al. [7] and McEwen et al. [8] on cosmic ray impacts provide the empirical foundation for our correlated burst noise model. Our contribution is the integration of these findings into a quantitative reachability framework that computes the logical error floor imposed by correlated noise.

IX. Conclusion and Future Work

We have presented a noise reachability analysis framework that unifies noise classification, multi-code QEC evaluation, and correction recommendation into a single automated pipeline. The framework introduces a twelve-type noise taxonomy across three source categories, evaluates each noise sub-type against six QEC codes at multiple distances (720 total evaluations), and produces severity-classified reachability assessments with quantitative suppression factors. An automated recommendation engine applies decision-tree logic to select optimal codes, and 24 source-specific compensating controls address errors that exceed correction capability.

The key findings from our evaluation are:

1) Standard noise regimes are well-served by existing codes. At $p_{\text{phys}} = 10^{-3}$, all twelve noise sub-types are correctable by the surface code at distance $d = 5$ with suppression factors exceeding 10^3.
2) Noise characteristics determine optimal code choice. Under elevated noise with moderate bias, the color code at higher distance can outperform the surface code due to its transversal Clifford gates, reducing overall resource costs despite a lower threshold.
3) Correlated noise creates uncorrectable floors. Cosmic ray bursts at rates above 10^{-4} impose a logical error floor that cannot be overcome by increasing code distance, requiring physical-level mitigations such as phonon traps.
4) Real-time analysis is achievable. The complete 720-evaluation pipeline executes in under 2 milliseconds, enabling interactive noise-aware code selection.

A. Future Work

Several directions for extending this work are under investigation:

Adaptive noise tracking. Integrate real-time noise estimation from syndrome data streams to continuously update the noise classification and trigger re-evaluation of code recommendations when noise characteristics change.

Machine learning decoders. Replace analytical logical error rate formulas with neural network decoders trained on platform-specific noise distributions, improving the accuracy of reachability assessments for hardware-specific noise correlations.

Concatenated and hierarchical codes. Extend the evaluation matrix to include concatenated code constructions (e.g., surface-code-encoded qubits within a Reed-Muller code) that can provide more efficient implementations of non-Clifford gates.

Hardware-aware code selection. Incorporate connectivity constraints, gate fidelity maps, and frequency collision data from specific quantum processors to restrict the code recommendation to implementable configurations.

Expanded noise taxonomy. Extend the twelve-type taxonomy to include emerging noise sources such as quasiparticle poisoning in superconducting qubits, heating in trapped-ion chains, and atom loss in neutral-atom arrays.

Multi-logical-qubit analysis. Extend reachability analysis from the single-logical-qubit setting to evaluate inter-logical-qubit error propagation in large-scale fault-tolerant circuits, including the impact of lattice surgery and code deformation operations.

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